CHE374 – Problem Set #7 (Saturday 26th October 2024)

Question 1) A machine purchased for \$92,000 has a depreciable price of 4 years. It will have an expected salvage value of \$19,000 at the end of the depreciable life. What would be the book value of the asset in each of the 4 years if the following depreciation methods were used.

Variables: Purchase Price (PP) = \$92,000
Depreciation Life = 4 years
Salvage Value (SV) = \$19,000
Total Depreciation = PP − SV = \$92,000 − \$19,000 = \$73,000

(A) Straight Line Depreciation

$$\text{Annual Depreciation (AD)} = \frac{\text{Total Depreciation}}{\text{Life}} = \frac{\$73{,}000}{4} = 18250 \ \$/\text{year}$$

At the end of

Year 0: BV = PP = \$92,000
Year 1: BV = PP − AD(t) = \$92,000 − (18250 × 1) = \$73,750 ✓
Year 2: BV = PP − AD(t) = \$92,000 − (18250 × 2) = \$55,500 ✓
Year 3: BV = PP − AD(t) = \$92,000 − (18250 × 3) = \$37,250 ✓
Year 4: BV = PP − AD(t) = \$92,000 − (18250 × 4) = \$19,000 ≡ Salvage Value ✓

(B) Declining Balance Depreciation → $\text{rate (d)} = 1 - \sqrt[\text{Life}]{\frac{SV}{PP}} = 1 - \sqrt[4]{\frac{\$19{,}000}{\$92{,}000}} = 0.32587 \approx 32.59\%$

At the end of

Year 0: BV = PP = \$92,000
Year 1: $BV = PP(1-d)^t = \$92{,}000(1-0.32587)^1 = \$62019.96$ ✓
Year 2: $BV = PP(1-d)^t = \$92{,}000(1-0.32587)^2 = \$41809.52$ ✓
Year 3: $BV = PP(1-d)^t = \$92{,}000(1-0.32587)^3 = \$28185.04$ ✓
Year 4: $BV = PP(1-d)^t = \$92{,}000(1-0.32587)^4 = \$19{,}000.39$ ✓

(C) Fixed rate depreciation with d = 25% (similar to declining balance but rate is pre specified:

At the end of:

Year 0: BV = PP = \$92,000
Year 1: $BV = PP(1-d)^t = \$92{,}000(1-0.25)^1 = \$69{,}000.00$ ✓
Year 2: $BV = PP(1-d)^t = \$92{,}000(1-0.25)^2 = \$51750.00$ ✓
Year 3: $BV = PP(1-d)^t = \$92{,}000(1-0.25)^3 = \$38812.50$ ✓
Year 4: $BV = PP(1-d)^t = \$92{,}000(1-0.25)^4 = \$29109.38$ ✓

(d) Double Declining Balance $\left(d = \frac{2}{\text{Life}} = \frac{2}{4} = 0.5\right)$

At the end of:

Year 0: BV = PP = \$92,000
Year 1: $BV = PP(1-d)^t = \$92{,}000(1-0.5)^1 = \$46{,}000.00$
Year 2: $BV = PP(1-d)^t = \$92{,}000(1-0.5)^2 = \$23{,}000.00$
Year 3: $BV = PP(1-d)^t = \$92{,}000(1-0.5)^3 = \$11{,}500.00$
Year 4: $BV = PP(1-d)^t = \$92{,}000(1-0.5)^4 = \$5750{,}00$

(e) Sum of the Years' Digits (SOYD = 1+2+3+4 = 10)

Depreciation Rate (DR) $= \frac{Life - t + 1}{SOYD}$ $\qquad D_t = \frac{N-t+1}{SOYD}(PP - SV)$ } Depreciation for year t

PP − SV = \$73,000

Year 0: BV = \$92,000

Year 1: $DR = \frac{4-1+1}{10} = 0.4$ $\qquad$ BV = \$92,000 − (0.4·73,000) = \$62,800.00

Year 2: $DR = \frac{4-2+1}{10} = 0.3$ $\qquad$ BV = \$92000 − [(0.4+0.3)73,000] = \$ 40900.00

Year 3: $DR = \frac{4-3+1}{10} = 0.2$ $\qquad$ BV = \$92,000 − [(0.4+0.3+0.2)73,000] = \$ 26,300.00

Year 4: $DR = \frac{4-4+1}{10} = 0.1$ $\qquad$ BV = \$92,000 − [(0.4+0.3+0.2+0.1) 73,000] = \$ 19,000.00

(f) Units of production assuming the following units production rates.

Year	Units
0	—
1	37,000
2	37,000
3	32,000
4	30,000

$D_t = \frac{\text{Production in Year } t}{\text{Lifetime Production}} (PP - SV)$

Life time production = 37000 + 37000 + 32,000 + 30,000
= 136,000

Depreciation Rate $= \frac{(PP-SV)}{\text{Lifetime Production}} = \frac{73,000}{136,000} = 0.536764$ \$ per unit

BV = PP − (Total production at the end of Year t) x DR

At the end of year:

Year 0: BV = \$92,000

Year 1: BV = \$92,000 − (37,000) (0.536764) = \$ 72,139.732 ✓

Year 2: BV = \$ 92,000 − (37,000 + 37000) (0.536764) = \$ 52279.464 ✓

Year 3: BV = \$ 92,000 − (37,000 + 37000 + 32,000) (0.536764) = \$ 35103.016 ✓

Year 4: BV = \$ 92,000 − (37,000 + 37000 + 32,000 + 30,000) (0.536764) = \$ 19000.096 ✓

Question 2) Determine the book value of a machine purchased for \$210,000 with an estimated life of 20 years and a salvage value of \$10,000, at the end of year 6 using the following methods (calculate these values by directly applying the formulas):

In each of the below, what was the depreciation amount booked at year 6?

Variables: Purchase Price (PP) = \$210,000
Salvage Value (SV) = 10,000
Depreciation Life = 20 years
Total Depreciation = PP − SV = \$210,000 − \$10,000 = \$200,000

(a) Straight Line Depreciation

Annual Depreciation $= \frac{\text{Total Depreciation}}{\text{Life}} = \frac{\$200,000}{20} = \$10,000$ /year

Book Value at end year 6 = PP − (AD × t)
= \$210,00 − (10,000 × 6)
= \$150,000

Depreciation Amount = \$ 10,000.

(b) Declining Balance Depreciation. $\left(d = 1 - \sqrt[Life]{\frac{SV}{PP}} = 1 - \sqrt[20]{\frac{\$10,000}{\$210,000}} = 0.1412059\right.$

At the end of year 6: $BV = PP(1-d)^t = \$210,000(1-0.1412059)^6$
$= \$84246.84$

Depreciation Amount paid during year 6 $= BV_6 - BV_5$

$$\text{Amount} = PP(1-d)^6 - PP(1-d)^5 = PP(1-d)^5(d)$$
$$= \$210000(1-0.1412059)^6(0.1412059)$$
$$= \$13852.16$$

(c) Fixed Rate Depreciation with $d = 20\%$.

At the end of year 6: $BV_6 = PP(1-d)^t = \$210{;}000\,(1-0.2)^6 = \$55050.24$

Depreciation Amount $= BV_t - BV_{t-1} = BV_6 - BV_5$
$= PP(1-d)^6 - PP(1-d)^5 = PP(1-d)^5\,d$
$= \$210,000(1-0.2)^5 \times 0.2 = \$13762.56$

(d) Double Declining Balance $\left(d = \frac{2}{Life} = \frac{2}{20} = \frac{1}{10} = 0.1\right)$

At the end of year 6: $BV_6 = PP(1-d)^6 = \$210,000(1-0.1)^6 = \$111602.61$

Depreciation Amount $= BV_t -$ $= BV_6 - BV_5$
$= PP(1-d) - PP(1-d)^5 = PP(1-d)^5 d$
$= \$210,000(1-0.1)^5\ 0.1$
$= \$12400.29.$

Question 3) The purchase price for a pump was \$145,000, 6 years ago. The current market value is estimated to be \$57,000. What would you estimate the market value to be in 2 years from now based on:

Variables → Purchase Price = \$145,000, Current Price = \$57,000, Life = 6 years.

(a) Straight Line Depreciation

$$\text{Annual Depreciation} = \frac{\$145,000 - \$57,000}{6\text{ (years)}} = \frac{\$88,000}{6} = \$14666.67$$

BV_{+2} = Current Price $- AD \times 2$
$= \$57,000 - (14666.67 \times 2) = \$27666.67$

(b) Declining Balance Depreciation

$$\text{rate} = d = 1 - \sqrt[Life]{\frac{\text{Current Price}}{\text{Original Price}}} = 1 - \sqrt[6]{\frac{\$57,000}{\$145,000}} = 0.1441102\ldots$$

$BV_{+2} = PP(1-d)^t = 57,000(1-0.1441102)^2 = \$41755.19$

(c) Why would it not make sense to use the double declining balance method here? In case of Straight line and Declining Balance we can set the depreciation rate and since we have the initial price and estimated current market rate, we can utilize this data for SL and DB methods. For double declining balance, we do not include the estimated price depreciation value. Note that in this question we are trying to estimate the actual market value, not book value.

Question 4) A machine purchased for \$110,000 has a depreciable life of 4 years. It will have an expected salvage value of \$25,000 at the end of depreciable life. What would be the book value of the asset in each of the 4 years if the following depreciation methods were used:

Variables:
- Basis (Purchase Prise), B = \$110,000
- Salvage Value, S = \$25,000
- Depreciation Life = 4 years
- Total Depreciation = B − S = \$110,000 − \$25,000 = \$85,000

(a) Straight line Depreciation

$$\text{Annual Depreciation (AD)} = \frac{\text{Total Depreciation}}{\text{Life}} = \frac{\$85{,}000}{4\text{ years}} = \$21{,}250/\text{year}$$

At the end of
Year 0: $BV_0 = B = \$110{,}000$
Year 1: $BV_1 = B - (AD \times t) = \$110{,}000 - (21250 \times 1) = \$88{,}750.00$
Year 2: $BV_2 = B - (AD \times t) = \$110{,}000 - (21250 \times 2) = \$67{,}500.00$
Year 3: $BV_3 = B - (AD \times t) = \$110{,}000 - (21250 \times 3) = \$46{,}250.00$
Year 4: $BV_4 = B - (AD \times t) = \$110{,}000 - (21250 \times 4) = \$25{,}000.00$

(b) Declining Balance Depreciation

$$\text{rate} = d = 1 - \sqrt[\text{Life}]{\frac{S}{B}} = 1 - \sqrt[4]{\frac{\$25{,}000}{\$110{,}000}} = 0.309543$$

At the end of
Year 0: $BV_0 = B = \$110{,}000$
Year 1: $BV_1 = B(1-d)^t = \$110{,}000(1-0.309543)^1 = \$75950.27$
Year 2: $BV_2 = B(1-d)^t = \$110{,}000(1-0.309543)^2 = \$52440.39$
Year 3: $BV_3 = B(1-d)^t = \$110{,}000(1-0.309543)^3 = \$36207.84$
Year 4: $BV_4 = B(1-d)^t = \$110{,}000(1-0.309543)^4 = \$25{,}000.00$

(c) Fixed Rate Depreciation with d = 35% (Similar to declining balance but rate is pre-specified)

At the end of
Year 0: $BV_0 = B = \$110{,}000$
Year 1: $BV_1 = B(1-d)^t = \$110{,}000(1-0.35)^1 = \$71{,}500.00$
Year 2: $BV_2 = B(1-d)^t = \$110{,}000(1-0.35)^2 = \$46{,}475.00$
Year 3: $BV_3 = B(1-d)^t = \$110{,}000(1-0.35)^3 = \$30208.75$
Year 4: $BV_4 = B(1-d)^t = \$110{,}000(1-0.35)^4 = \$19635.69$

(d) Double Declining Balance. $\left(\text{rate} = d = \frac{2}{\text{Life}} = \frac{2}{4} = 0.5\right)$

At the end of
Year 0: $BV_0 = B = \$110{,}000$
Year 1: $BV_1 = B(1-d)^t = \$110{,}000(1-0.5)^1 = \$55{,}000.00$
Year 2: $BV_2 = B(1-d)^t = \$110{,}000(1-0.5)^2 = \$27{,}500.00$
Year 3: $BV_3 = B(1-d)^t = \$110{,}000(1-0.5)^3 = \$13{,}750.00$
Year 4: $BV_4 = B(1-d)^t = \$110{,}000(1-0.5)^4 = \$6{,}875.00$

(e) Sum of Years' Digits $\left(SOYD = \sum_{i=0}^{\text{Life}} i = 1+2+3+4 = 10\right)$ $B - S = \$85{,}000$

$$\text{Depreciation Rate}_t = \frac{N-t+1}{SOYD} \qquad BV = B - \left(\sum_{i=0}^{t} DR_i\right)(B-S)$$

At the end of: Year 0: $BV_0 = B = \$110{,}000$

Year 1: $DR_1 = \frac{4-1+1}{10} = 0.4 \rightarrow BV_1 = \$110{,}000 - (0.4)85{,}000 = \$76{,}000.00$

Year 2: $DR_2 = \frac{4-2+1}{10} = 0.3 \rightarrow BV_2 = \$110{,}000 - (0.4+0.3)85{,}000 = \$50{,}500.00$

Year 3: $DR_3 = \frac{4-3+1}{10} = 0.2 \rightarrow BV_3 = \$110{,}000 - (0.4+0.3+0.2)85{,}000 = \$33{,}500.00$

Year 4: $DR_4 = \frac{4-4+1}{10} = 0.1 \rightarrow BV_4 = \$110{,}000 - (0.4+0.3+0.2+0.1)85000 = \$25{,}000$

(f) Units of Production assuming the following units production rate.

Year	Units
0	—
1	80,000
2	65,000
3	50,000
4	35,000

Depreciation Amount in year t $\left\{ D_t = \frac{\text{Production in Year } t}{\text{Lifetime Production}} (B-S) \right.$

Lifetime Production $= \$80{,}000 + \$65{,}000 + \$50{,}000 + \$35{,}000$
$= 230{,}000$ units

Depreciation Rate $= \frac{B-S}{\text{Lifetime Production}}$

$BV_t = B - (\sum_{i=0}^{t} P_i) \times DR$ Production in Year i

Depreciation Rate $= \frac{\$85{,}000}{230{,}000 \text{ units}} \doteq \$0.36956$ per unit

At the end of:

Year 0: $BV_0 = B = \$110{,}000$

Year 1: $BV_1 = B - P_1 DR = \$110000 - (80{,}000 \times 0.36956) = \$80434.78$

Year 2: $BV_2 = B - (P_1 + P_2) DR = \$110{,}000 - (80{,}000 + 65000) \times 0.36956 = \$56413.04$

Year 3: $BV_3 = B - (P_1 + P_2 + P_3) DR = \$110{,}000 - (80{,}000 + 65{,}000 + 50{,}000) \times 0.36956 = \$37934.78$

Year 4: $BV_4 = B - (P_1 + P_2 + P_3 + P_4) DR = \$110{,}000 - (80{,}000 + 65{,}000 + 50{,}000 + 35{,}000) \times 0.36956 = \$25{,}000$

Question 5) A 3-D printer has a useful life of 7 years and an initial selling price of \$ 23,000. The salvage value after 7 years is \$ 4,000. The market value of a 3-year old printer is \$ 12,000. Calculate the book value after 4 years and the depreciation amount for the 5th year using the following depreciation methods:

(a) Straight Line Depreciation

Annual Depreciation $= \frac{\$23{,}000 - \$4{,}000}{7 \text{ years}} = \$2714.29$ per year

At the end of year 4: $BV_4 = B - (AD \cdot t) = \$23{,}000 - (2714.29 \times 4) = \$12142.86$

Depreciation Amount for Year 5 $= \$2714.29$

(b) Declining Balance Depreciation

rate: $d = 1 - \sqrt[\text{life}]{\frac{S}{B}} = 1 - \sqrt[7]{\frac{\$4{,}000}{\$23{,}000}} = 0.2211101\ldots$

At the end of year 4: $BV_4 = B(1-d)^t = \$23{,}000\,(1 - 0.221101)^4$
$= \$8465.10$

Depreciation Amount for Year 5: $D_5 = BV_5 - BV_4$

$D_5 = B(1-d)^5 - B(1-d)^4 = B(1-d)^4 (d)$

$= \$23,000\,(1-0.221101)^4 \times 0.221101$

$= \$1871.72$

(c) Sum of Digits $\left(SOYD = \sum_{i=1}^{Life} i = 1+2+3+4+5+6+7 = 28\right)$ $BV_t = B - \left(\sum_{i=1}^{t} DR_i\right)(B-S)$

$DR_t = \frac{N-t+1}{SOYD}$ $DR_1 = \frac{7}{28}$; $DR_2 = \frac{6}{28}$; $DR_3 = \frac{5}{28}$; $DR_4 = \frac{4}{28}$

Book Value of Year 4 : $BV_4 = \$23,000 - \left(\frac{7}{28} + \frac{6}{28} + \frac{5}{28} + \frac{4}{28}\right)(19,000) = \$8071.43$

Depreciation Amount for Year 5 $= \frac{N-t+1}{SOYD}(B-S) = \frac{7-5+1}{28}(\$23,000 - \$4,000)$

$= \$2035.71$

(d) Unit of Production based on the following expected production of 3D projects

Year	Units
0	—
1	50
2	60
3	40
4	20
5	10
6	15
7	5

$D_t = \frac{\text{Production in Year } t}{\text{Lifetime Production}}(B-S)$

Lifetime Production $= 5+15+10+20+40+60+50 = 200$ units

Depreciation Amount in Year 5 $= \frac{10}{200}(\$23,000 - \$4,000)$

$= \frac{1}{20}(19,000) = \$950.00$

Depreciation Rate $= \frac{B-S}{\text{Lifetime Production}} = \frac{\$19,000}{200 \text{ units}} = 95$

Book Value $= PP - \left(\sum_{i=0}^{t} P_i\right) \times DR$

For Year 4 $= \$23,000 - (50+60+40+20) \times 95 = \$6850$

Question 6) SNQ Inc. purchased a production line with expected life of 10 years for \$ 2,500,000 at the beginning of 2012-2013 fiscal year. The current market value of the production line is estimated at \$ 2,300,000. Assuming a salvage value of \$ 200,000, calculate the book value of the production line at the end of the 2015-2016 fiscal year and the depreciation amount for that year under the following methods:

Variables: Initial Value (B) = \$ 2,500,000
Salvage Value (S) = \$200,000
Depreciable Life (N) = 10
Total Depreciation = \$2,500,000 − \$ 200,000 = \$2,300,000

(a) Straight Line Depreciation

Annual Depreciation $= \frac{\text{Total Depreciation}}{\text{Depreciation Life}} = \frac{\$2,300,000}{10} = \$230,000/\text{year}$

Depreciation amount 2015-2016 fiscal year = \$230,000

At the end of 2015-2016 fiscal year (4th year): $BV_4 = \$2{,}500{,}000 - (230{,}000 \times 4) = \$1{,}580{,}000$

(b) Declining Balance Depreciation

$$rate = d = 1 - \sqrt[N]{\frac{S}{B}} = 1 - \sqrt[10]{\frac{\$200{,}000}{\$2{,}500{,}000}} = 0.22320039\ldots$$

At the end of 2015-2016 fiscal year:

$$BV_4 = B(1-d)^4 = \$2{,}500{,}000\,(1-0.22320039)^4 = \$910282.10$$

$$\text{Depreciation Amount} = BV_4 - BV_3 = B(1-d)^4 - B(1-d)^3 = B(1-d)^3\,d$$

$$= \$2{,}500{,}000\,(1-0.22320039)^3(0.22320039) = \$261554.35$$

(c) Double Declining Balance Depreciation $\left(d = \frac{2}{N} = \frac{2}{10} = 0.2\right)$

At the end of 2015-2016 fiscal year: $t=4$

$$\text{Book Value (Year 4)} = B(1-d)^4 = \$2{,}500{,}000(1-0.2)^4 = \$1{,}024{,}000$$

$$\text{Depreciation Amount} = BV_4 - BV_3 = B(1-d)^4 - B(1-d)^3 = B(1-d)^3\,d$$

$$= \$2{,}500{,}000\,(1-0.2)^3(0.2) = \$256{,}000$$

(d) Sum of Years' Digit. $\left(SOYD = \sum_{i=1}^{N} i = 1+2+3+4+5+6+7+8+9+10 = 55\right)$

$$\text{Book Value (Year T)} = BV_T = B - \left(\sum_{i=1}^{t} DR_t\right) \times \underbrace{(B-S)}_{B-S = 2{,}300{,}000}$$

$$DR_t = \frac{N-t+1}{SOYD} \rightarrow DR_1 = \frac{10}{55}\ ;\ DR_2 = \frac{9}{55}\ ;\ DR_3 = \frac{8}{55}\ ;\ DR_4 = \frac{7}{55}$$

At the end of 2015-2016 fiscal year (Year 4):

$$BV_4 = \$2{,}500{,}000 - \left(\frac{10+9+8+7}{55}\right) \times 2{,}300{,}000 = \$1{,}078{,}181.82$$

$$\text{Depreciation Amount} = \frac{N-t+1}{SOYD}(B-S)$$

Depreciation Amount for 2015-2016 fiscal year

$$D = \frac{10-4+1}{55}(\$2{,}500{,}000 - \$200{,}000)$$

$$= \frac{7}{55}(\$2{,}300{,}000) = \$292{,}727.27$$